Investigation of laminar flow frictional losses in a large-hydraulic-diameter pipe and annulus

P Suresh Kumar*
Department of Ocean Engineering and Naval Architecture, Indian Institute of Technology, Kharagpur 721302, India

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Abstract: In the present work an experimental study has been carried out to study the friction factor variation with Reynolds number for laminar flow in a large-hydraulic-diameter pipe and annulus. It is found that for low Reynolds numbers the friction factors are large than those reported in the literature for small-hydraulic-diameter pipe and annulus. Large hydrostatic pressure variation along the circumferential direction causes a different flow pattern in a large-hydraulic-diameter duct and may be why the present results do not match those reported in the literature. A correlation has been proposed in the present paper which is being developed using the present experimental results for both pipe and annulus to correlate the friction factor as a function of Reynolds number and a newly defined Jaga number *Jg*. An analysis has been carried out using the currently developed friction factor correlations to study how the friction factor will vary for different fluids and different diameters of the pipe and annulus. It is observed that, for high Reynolds numbers ($Re > 100$), small-hydraulic-diameter duct and fluids with a large kinematic viscosity, the present correlations show good agreement with the results reported in the literature.

Keywords: hydrostatic pressure, longitudinal flow, secondary flow

1 INTRODUCTION

The friction losses in pipes are increasingly often calculated using the White–Colebrook formula (as employed by Nikuradse, Moody, Prandtl, and Von Karman) instead of the empirical formula utilized by Chezy, Manning, Hazen Williams, Flammant, and others. The White–Colebrook formula is based on the mixing length turbulent model (due to Prandtl and Von Karman), the velocity defect law (universal logarithmic velocity law described by Prandtl), and extensive measurements by Nikuradse. The friction factor formula has been verified by numerous measurements [**1**]. The work of Zagarola [**2**] is well known as the 'Princeton experiment' and is considered to be the new benchmark after the work of Nikuradse. It showed good agreement with the universal logarithmic law. Although the White–Colebrook formulation (for turbulent flow) and the Moody formula (for laminar flow) for smooth pipes have been validated by different large experimental data sets, very few or none of these sets has been obtained in large-diameter pipes (50 mm or greater) for a small-Reynolds-number range (laminar flow region). There are some indications in the literature that the unconditional validity of the White–Colebrook formula and the Moody equation for very-large-diameter smooth pipes may be questionable. Such very-large-diameter pipes are used for example as pressurized trunk sewers, penstocks, and filling and emptying devices for navigation locks. Small-Reynolds-number flow in a large-diameter pipe and annulus is important in situations where flow occurs due to natural convection as in solar water heaters. It appears indeed from measurements that sometimes, in particular for large Reynolds numbers, smooth pipes may have smaller friction factors than those obtained from the White–Colebrook formula. In a way they seem to be 'smoother than smooth' or even have negative roughness [**3**]. As the White–Colebrook equation is generally accepted, so it seems inappropriate to publish such results and rather to blame measurement errors for

these deviations. However, such measurements have been reported by several researchers such as Ackers [**4**], Barr [**3**], Levin [**5**], and Burke [**6**]. Nikuradse's smooth turbulent flow data and especially those data obtained in one particular 100 mm pipe are used to support the logarithmic smooth law, which is in turn used to discount data which is not in agreement with it [**3**]. Levin [**5**] concluded that, with the smoothest surface tested, the data lay significantly below the accepted logarithm smooth line. The experimental results [**3**] seemed to be much closer to the Blasius law than to the smooth pipe law obtained by Nikuradse. Burke's [**6**] experiments show that, for a 3.124 m pipe, many of the friction factors measured are 10 per cent lower than the logarithmic law. So far, this issue has never been addressed in the literature.

In the literature, evidence is found for measured friction factors in pipes, which are smaller than would be expected in a perfectly smooth pipe. This occurs in particular at high Reynolds numbers and in large-diameter pipes. These apparently impossible results cannot be explained by errors in the measurements. From both experiments and numerical simulation it is known that rotation causes a certain degree of laminarization of an initially turbulent pipe flow and accordingly a reduction in friction factor. Recently both experimental and numerical work (direct numerical simulation of the Navier–Stokes equation) by Orlandi [**7**] and Orlandi and Fatica [**8**] has been carried out to study flows in a pipe rotating around its axis. It has been found that, in an axially rotating pipe flow, so-called 'laminarization' occurs; i.e. the turbulent flow becomes, in particular close to the rotating wall, similar to a laminar flow when the rotation rate increases, resulting in the deformation of the axial velocity profile into a shape similar to the laminar profile. This leads to a drag reduction and subsequent decrease in the friction factor up to 50 per cent [**9–12**].

The aforementioned investigations conclude that the flow pattern for which the classical friction factor laws are developed is not suitable for large-diameter pipes as in a large-diameter pipe the flow pattern is different from that in a small-diameter pipe. It apparently does not make any difference whether the rotation is imposed by a rotating pipe wall or as an initial condition. If, however, a flow had obtained for some reason (e.g. the passage of a bend, a flow regulator on pipe joints, which are present in almost all experimental studies) a certain amount of rotation, it may persist for a long time; in particular, in a large-hydraulic-diameter pipe it may persist for a longer distance along the length of the pipe. This may be the explanation why in a number of circumstances during experiments a moderate amount of rotation may be present in a straight pipe and is the reason why friction factors obtained for large-diameter pipe do not agree with the equations presented by White and Colebrook. The deviation from the classical friction laws was only observed in smooth large-diameter pipes [**5**].

The above finding on friction factors in the turbulent flow region for a large-hydraulic-diameter pipe motivated the present study. The present work is an investigation in the laminar flow region to determine whether the friction factors in a large-hydraulic-diameter pipe and annulus are different or not from the results that exist in the literature for a small-hydraulic-diameter pipe and annulus (the Moody chart for the pipe, and results obtained by Olson and Wright [**13**] for the annulus). A series of experiments have been carried out in the laminar flow region to study friction factor variation in a pipe and annulus with a large hydraulic diameter. It is found that the friction factors in a large-hydraulic-diameter pipe and annulus are quite different from those in the literature for a small-hydraulic-diameter pipe and annulus. The present paper provides the probable reasons for this deviation of friction factors from those proposed in classical theories. Also an attempt has been made to correlate the friction factor as a function of the Reynolds number dealing with a regular longitudinal flow pattern and a new non-dimensional number dealing with the secondary flow effects in large-hydraulic-diameter duct-flow.

2 CORRELATION FORMULATION

Reich and Beer [**10**] experimentally studied a pipe of 50 mm diameter with air as a working fluid in the turbulent flow region and observed that, if the flow is initially turbulent, the pressure loss decreases with increase in rotational effects. The rotational effects were found to suppress turbulence in the flow, which as a result led to a reduced hydraulic loss. It is clear from the previous evidence that the flow in a large-diameter long straight pipe is not stable and a helical flow pattern develops as a superposition of the regular longitudinal flow and a secondary flow pattern (transmission of inlet perturbation and small perturbation from pipe joints), similar to the secondary flows in open channels. This is analogous to the development of bed forms in rivers and on seabeds, the development of roll waves in a supercritical open channel flow [**14**, **15**] and even the meandering of a river pattern [**16**, **17**].

In an open channel or in an ocean generally the wind force acts as an excitation force for the generation of harmonic wave motion where the gravity (hydrostatic pressure or weight of the water) acts as a restoring force. In case of flow through a pipe and annulus the momentum loss due to obstacles acts

as an excitation force for the harmonic secondary flow generation where the hydrostatic pressure acts as a restoring force and the force due to viscosity of fluid acts as a damping force. For a large-diameter pipe and a fluid of low kinematic viscosity (the ratio of the dynamic viscosity to the density of the fluid) the restoring force (hydrostatic pressure) for the harmonic secondary flow is large due to the large radial head for a high-density fluid and at the same time the damping force (the force due to fluid viscosity) is small because the dynamic viscosity of the fluid is low. This is the reason why the initial secondary flow exists for a longer duration and distance in a large-diameter pipe and annulus for a fluid of low kinematic viscosity.

The rotation effects due to secondary flow have a significant effect on the flow pattern in a large-diameter pipe and annulus. The flow pattern has a significant effect on the pressure drop, and the pressure drop ultimately depends on the friction factor. Hence the laminar flow friction factors in a large-diameter pipe and annulus must be correlated such that it will take care of the secondary flow effects, in addition to the wall friction. It is well known that the pressure drop in laminar flow is very small but, if somehow the laminar flow has a superimposed secondary flow, as is the case in large-diameter pipe and annulus, then the pressure drop may increase significantly. Any rise in pressure drop subsequently leads to higher friction factors. In open channel flow or wave motion in an ocean the hydrostatic effect (gravity effect) is dealt with using the well-known Froude number, which is the ratio of the dynamic pressure to the hydrostatic pressure (or body force). In those cases generally the flow has a free surface and is considered as an ideal inviscid flow, because the velocity gradients are negligibly small whereas, for internal flows where the flow has no free surface, the velocity gradients are significantly high. Hence viscous effects must be taken into account for an enclosed flow.

In the present work, water is taken as the working fluid, which has a low kinematic viscosity. Hence according to the above reasons the secondary flow effects must play a vital role in the flow pattern. To deal with this problem in internal flows a new non-dimensional Jaga number Jg has been defined in the present work, which is the ratio of the dynamic pressure to the diffusion flux due to the secondary flow effect (enhanced by the large hydrostatic pressure) as given by

$$Jg = \frac{\rho V^2}{\mu\sqrt{g/D_o}} \tag{1}$$

where ρ is the density of the fluid, V is the average velocity based on ratio of the measured discharge to the flow area A, μ is the dynamic viscosity of the fluid, D_o is the inner diameter of the outer pipe for the annulus (as the hydrostatic pressure variation depends on the inner diameter of the outer pipe), which is simply the hydraulic diameter for a plane pipe, and g is the acceleration due to gravity. The newly defined non-dimensional Jaga number Jg is similar to the Reynolds number, which is the ratio of the dynamic pressure to the diffusion flux due to the regular longitudinal flow pattern as given by

$$Re = \frac{\rho V^2}{\mu(V/D_h)} \tag{2}$$

The only difference between new Jaga number Jg and the Reynolds number Re is that in the case of Jg the velocity gradient in the diffusion flux term is shifted from V/D_h to $\sqrt{g/D_o}$ as the secondary flow effects are dependent on the weight of the fluid.

All the experimental results for the total friction factor f_t obtained from the present experiments were correlated as a sum of two friction factor components; one is due to the regular longitudinal flow (f_w) and the other is due to secondary flow effects (f_h) according to

$$f_t = f_w + f_h \tag{3}$$

It is well known that the friction factor f_w in the pipe and annulus due to the regular longitudinal flow pattern is linearly proportional to the reciprocal of the Reynolds number Re. In the present experiments it is observed that the friction factor f_h due to the secondary flow pattern is linearly proportional to reciprocal of the newly defined Jaga number Jg. Therefore the total friction factor f_t can be written in non-dimensional form as

$$f_t = \frac{a}{Re} + \frac{b}{Jg} \tag{4}$$

where a and b are arbitrary dimensionless constants. The correlation developed in the present investigation based on the Jaga number Jg and the Reynolds number Re has shown good agreement with the existing results for a small-diameter pipe and annulus and liquids with a low kinematic viscosity $\nu = \mu/\rho$. The number Jg in the present investigation is found to be important for friction factor calculations in the duct flows where the ratio of the diffusion flux due to the secondary flow effect (enhance by large hydrostatic pressure) to the dynamic pressure is large, i.e. small values of Jg.

3 EXPERIMENTATION PROCEDURE

Figure 1 shows a schematic view of the experimental set-up. An overhead tank with 0.5 m^3 capacity serves as a constant-head reservoir and is used to discharge the test liquid to the test section through a regulating valve. The test section consists of concentric straight pipes made of Plexiglas, which are joined at regular intervals of 1.0 m by flanges. The inner diameter of the outer pipe is 50 mm and the annulus (flow passage) is 30 mm in the radial direction throughout. The small pipe is supported inside the large pipe centrally by means of a small concentric cylinder made of Plexiglas of 0.3 mm thickness and 0.5 mm length which is supported circumferentially in the large pipe every 2 m by means of three thin pins of diameter 0.25 mm. The test section is kept horizontal by aligning it by means of a spirit level.

Pressure taps were placed at distances of 4.0, 4.25, and 4.75 m along the flow direction from the inlet valve position. At each location, three pressure taps were placed circumferentially at angles of 0°, 120°, and 240° measured from the top along the clockwise direction (Fig. 2). The pressure taps are 40 mm long and made of cast iron with an inner diameter of 1 mm and an outer diameter of 3 mm. The joints of the pressure taps and the pipe are sealed with an adhesive to ensure no leakage. The pressure

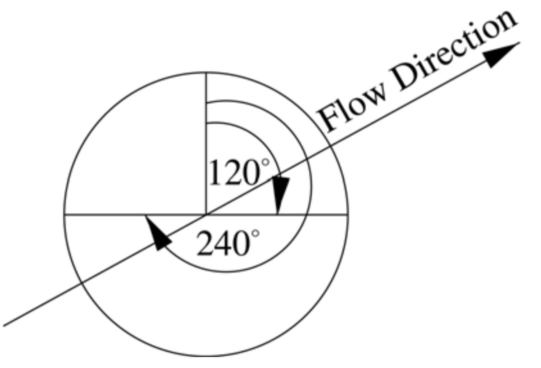

Fig. 2 Angular position for pressure taps

drop per unit length is found to be nearly same according to

$$\frac{\Delta P_{4.0} - \Delta P_{4.5}}{4.25 - 4.0} \approx \frac{\Delta P_{4.25} - \Delta P_{4.75}}{4.75 - 4.25} \tag{5}$$

Hence it is concluded that the flow is fully developed.

All the pressure readings are taken under isothermal conditions and a U-tube manometer with water as working fluid is used to measure the pressure drop at each probe location. The lowest value on the pressure measurement manometer is 0.25 mm. Water is used as the working fluid for the

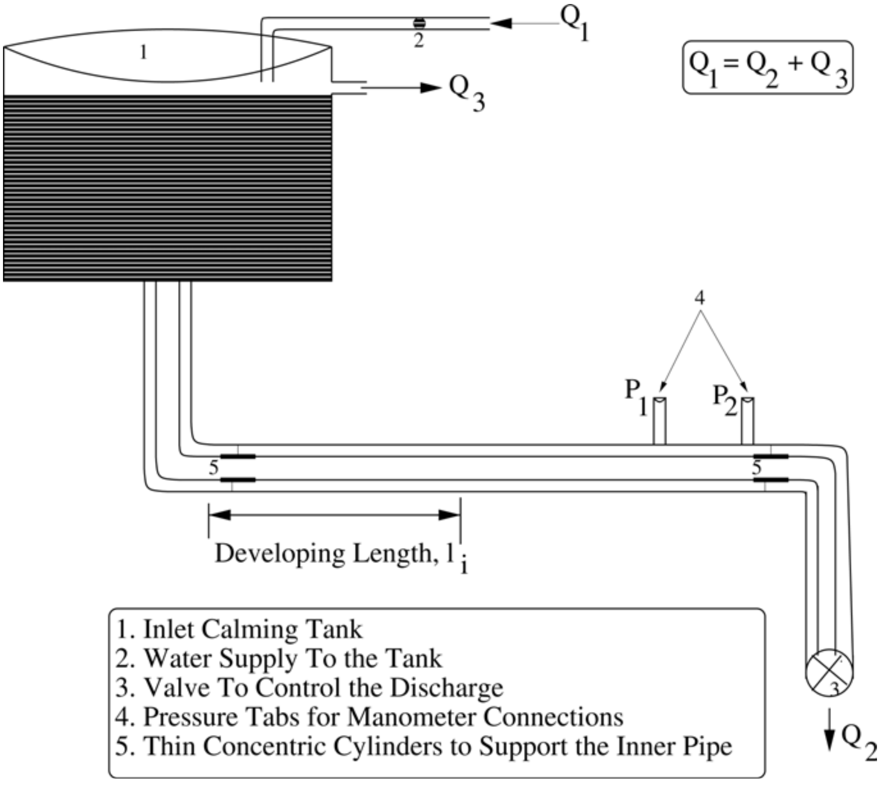

Fig. 1 Schematic view of the experimental set-up

entire study. Water is supplied to the pipe from a large reservoir, which is maintained at a constant head. The discharge through the pipe is measured by means of a calibrated jar and stopwatch.

Experimental data on pressure drop and discharge were collected under constant-head conditions. At least five sets of data were collected to ensure the repeatability of the experiments. The average of the five sets of readings is used for the present analysis. The developing length L_i for the laminar flow in an annulus is calculated using the correlation [18]

$$L_i = 0.056 D_h \, Re \tag{6}$$

where D_h is the hydraulic diameter and the Reynolds number Re is given by

$$Re = \frac{V D_h \rho}{\mu} \tag{7}$$

The friction factor f_t in the developed region is calculated from the average pressure drop readings of the three circumferential pressure taps at three streamwise locations (4.0, 4.25, and 4.75 m from the inlet valve location) as given by

$$f_t = \frac{2 h_l g D_h A^2}{l Q^2} \tag{8}$$

where h_l is the head loss noted from the manometer reading along the flow direction, g is the acceleration due to gravity, D_h is the hydraulic diameter, A is the area of the cross-section, and Q is the flow discharge rate through the test section. The Reynolds number Re and the new non-dimensional Jaga number Jg are calculated on the basis of the discharge rate Q through the test section.

An analysis of the uncertainties in the measurements by various instruments has been carried out by Kline and McClintontock [19]. The uncertainties in the calculated values of friction factor and Reynolds number are as follows:

(a) Reynolds number, ± 2.12 per cent;
(b) friction factor, ± 4.56 per cent.

Details of the experimental procedure followed and the uncertainties involved in the present study can be found in reference [20].

4 RESULTS AND DISCUSSION

In Figs 3 and 4 the variations in the Jaga number Jg with Reynolds number Re for an annulus with different values of the inner diameter D_o of outer pipe, keeping the ratio the same as that for the present experimental set-up ($D_i/D_o = 0.4$), and for a plane pipe for different hydraulic diameters D_h respectively

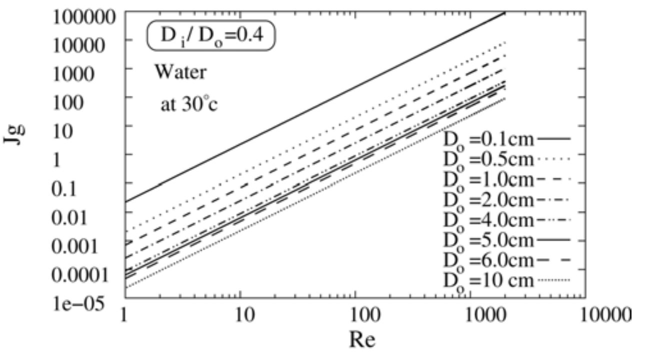

Fig. 3 Variations in Jg with Re in an annulus

are plotted. The diameter ratio for the annulus is kept constant because, according to Olson and Wright [13], this ratio affects the frictional losses significantly. It can be seen that Jg increases linearly with increasing Re. Further, Jg is large for small pipe diameters and large values of Re. The reason for this behaviour is that in these situations the dynamic pressure is quite high compared with the diffusion flux due to secondary flow effects. Hence, the high dynamic pressure restricts the existence of secondary flow effects and straightens the rotational velocity component. Flow in large-diameter pipes with high Reynolds numbers also leads to a high dynamic pressure which limits the rotational effects so that they exist only near the wall (where the streamwise velocity is very low) and this may be the reason that Reich and Beer [10] found that a 'smoother than smooth' condition or a wall with negative roughness exists in the turbulent flow region for a large-diameter pipe. According to equation (4) for the low-Reynolds-number range the total friction factor f_t has higher values when the hydraulic diameter of the pipe and annulus is large, because in that case the contribution due to Jg for the total friction factor is significant. This is the reason why the present results do not match those reported for small-hydraulic-diameter ducts by Moody for a plane pipe and by Olson and Wright [13] for an

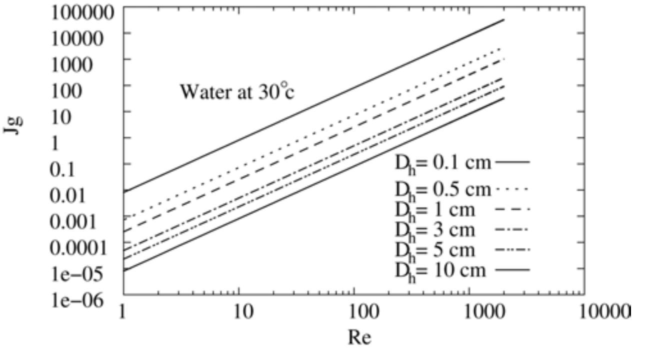

Fig. 4 Variations in Jg with Re in a plane pipe

annulus. The correlations developed in the present work yield values of friction factor in good agreement with those reported by Moody and by Olson and Wright [**13**] at high Reynolds numbers and for small value of hydraulic diameter.

The variations in *Jg* with *Re* in a plane pipe for different fluids are plotted in Fig. 5. It is clear that the value of *Jg* is small for fluids with low kinematic viscosities and vice versa. Hence the loss due to the secondary flow effects is large for the fluids with low kinematic viscosities. Therefore it can be concluded that the old friction factor results correlated in terms of the Reynolds number only for a particular liquid will not be applicable to another liquid when the difference between the kinematic viscosities of two fluids is large and the hydraulic diameter of the duct is large. Arguments on similar grounds can be presented for an annulus as well.

The present results for the variations in total friction factor with the Reynolds number in a large-hydraulic-diameter annulus and a smooth pipe are shown in Fig. 6, which also shows the comparison of data obtained from the correlations developed in the present work with the present measurements. The correlation data are in good agreement with the measurements (maximum error of 1.0 per cent). The correlations developed from the present experimental data for friction factor are as follows: for an annulus,

$$f = \frac{75}{Re} + \frac{6284.39}{Jg} \tag{9}$$

and, for a plane pipe,

$$f = \frac{26.79}{Re} + \frac{529346}{Jg} \tag{10}$$

It may be observed that the value of the total friction factor is higher for the plane pipe [equation (10)] than that for the annulus [equation (9)] in large-hydraulic-diameter ducts. The reason for

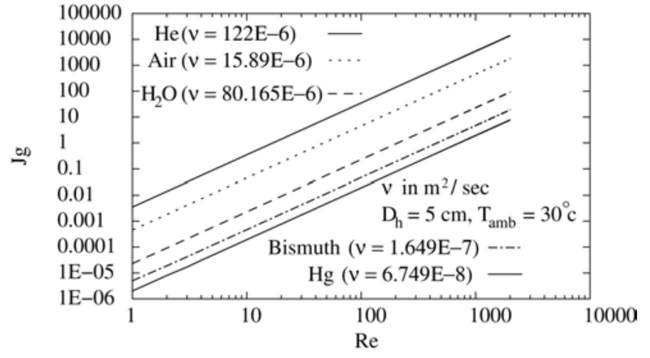

Fig. 5 Variations in *Jg* with *Re* for different fluids in a plane pipe

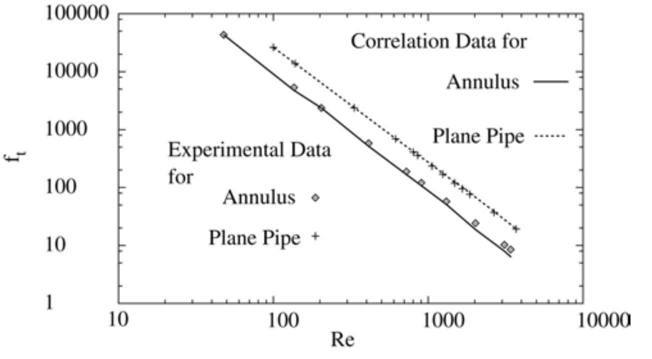

Fig. 6 Variations in f_t with *Re* in a large-hydraulic-diameter pipe and annulus

larger losses in the plane pipe than in the annulus is that in a large-hydraulic-diameter duct the wall friction losses are less important than secondary flow effects represented by the large hydrostatic pressure variation along the circumference. The more fluid there is in a given cross-section, the greater is the loss due to secondary flow. Hence in the plane pipe the values of friction factor are found to be high compared with those in the annulus. By comparing equations (9) and (10) it can be concluded that the total friction factors in the plane pipe are more sensitive to the Jaga number *Jg* than in the annulus, because the value of b/Jg is quite high in the case of the plane pipe. The experimental data and correlated data for the total friction factor variation with *Jg* are plotted in Fig. 7.

Using equation (9) the variations in total friction factor with the Reynolds number in the annulus are plotted in Fig. 8 for different inner diameters D_o of the outer pipe keeping the ratio D_i/D_o fixed at 0.4 for helium and in Fig. 9 for water at 30 °C. From Figs 8 and 9 it is clear that the results using the present correlation are in good agreement with those reported by Olson and Wright [**13**] only when the outer diameter is small ($D_o \leqslant 0.5$ cm) and the Reynolds number is large ($Re \geqslant 100$). The reason

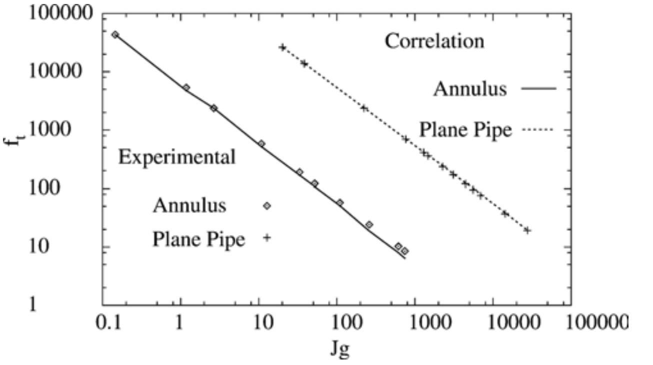

Fig. 7 Variations in f_t with *Jg* in a large-hydraulic-diameter pipe and annulus.

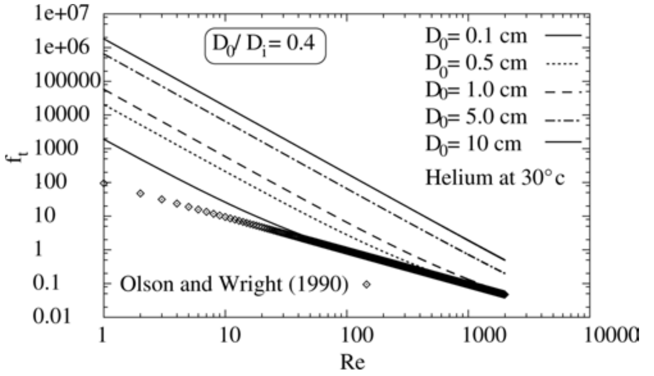

Fig. 8 Variations in f_t with Re in an annulus for different diameters D_o where the ratio D_i/D_o remains constant (and equal to 0.4) for helium

for this behaviour is that under this condition the dynamic pressure dominates over the diffusion flux due to secondary flow effects. Hence the high dynamic pressure nullifies the secondary flow effects enhanced by the hydrostatic pressure variation. However, it can be observed that the results for helium are in closer agreement than those for water because helium has a higher kinematic viscosity than water, which results in higher contribution from Jg to f_t in the case of water.

Using equation (10) the variations in total friction factor with the Reynolds number in a plane pipe are plotted in Fig. 10 for different diameters (for helium at 30 °C). The observation from this figure is similar to that of Fig. 8. The values of friction factor approaches the values reported in the Moody chart with a decrease in the hydraulic diameter and an increase in the Reynolds number.

The variations in the total friction factor with the Reynolds number for different liquids are plotted in Fig. 11 for a plane pipe keeping the diameter fixed at 3 cm. It can be seen from the figure that the values of the friction factor are quite high for liquids

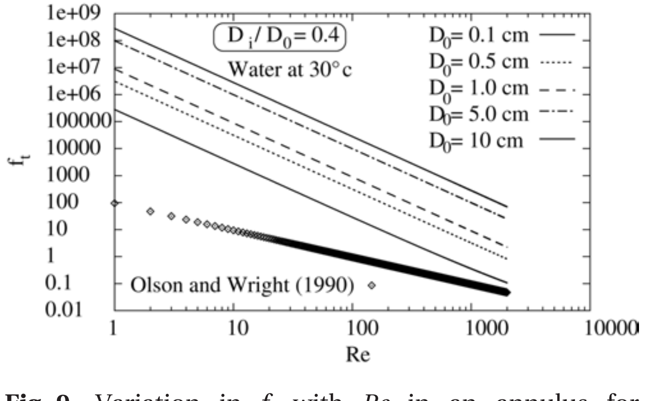

Fig. 9 Variation in f_t with Re in an annulus for different diameters D_o where the ratio D_i/D_o remains constant (and equal to 0.4) for water

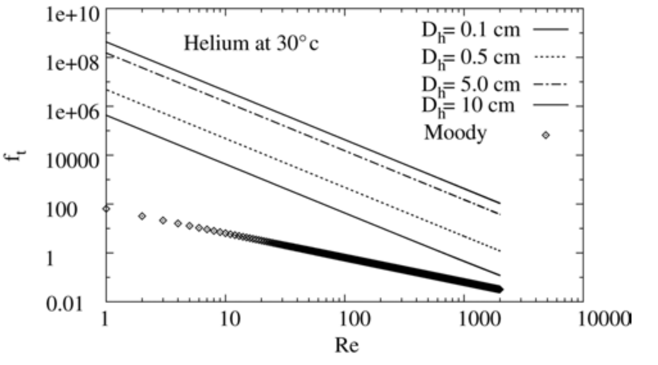

Fig. 10 Variation in f_t with Re, in a plane pipe for different diameters, using the properties of helium

with low kinematic viscosities because for low-kinematic-viscosity liquids the value of Jg is small; as a result the influence of the secondary flow on the friction factor is high.

5 CONCLUSIONS

The flow pattern in a small-hydraulic-diameter duct ($D_h \leqslant 30$ mm) is different from that in a large-hydraulic-diameter duct. In a large-hydraulic-diameter duct, the pressure loss is found to be dependent on both the general longitudinal flow pattern and the secondary flow pattern. The contribution of secondary flow effects to the friction factor increases with increase in the hydraulic diameter. The results obtained in the literature for a small-hydraulic-diameter pipe and annulus as a function of Reynolds number cannot be applicable to a large-diameter pipe and annulus because in a large-diameter pipe and annulus the flow is unstable and the effect of secondary flow must be considered to improve the accuracy in the friction factor

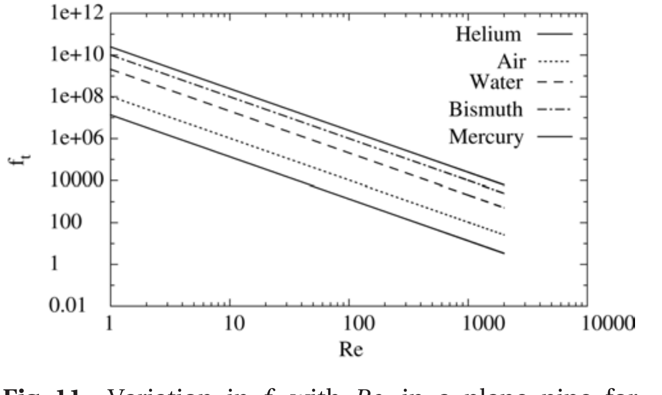

Fig. 11 Variation in f_t with Re, in a plane pipe for different liquids

calculation. The correlations developed here as a function of the Reynolds number *Re* and the newly defined Jaga number *Jg* show good agreement with those reported in literature when the hydraulic diameter is small, the Reynolds number is large and the fluid has a large kinematic viscosity.

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APPENDIX

Notation

a	arbitrary dimensionless constant
A	wetted area
b	arbitrary dimensionless constant
D_h	hydraulic diameter
D_i	outer diameter of the inner pipe for the annulus
D_o	inner diameter of the outer pipe for the annulus
f_h	friction factor due to secondary flow
f_t	total friction factor
f_w	friction factor due to wall friction
g	acceleration due to gravity
h_l	head loss
Jg	new non-dimensional Jaga number
l	length of pipe in the flow direction
L_i	developing length for the pipe
Q	discharge rate
Re	Reynolds number
T_{amb}	ambient temperature
V	average velocity
μ	dynamic viscosity of the fluid
ν	kinematic viscosity

